Modeling Earth-surface temperature extremes with physics-informed upper bounds for simulating worst possible cases

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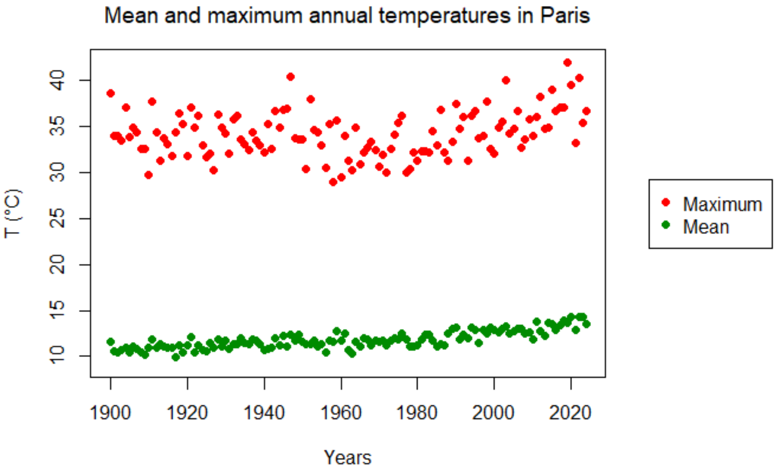
Context:

- Strong statistical and physical evidence for a finite land temperatures upper bound
- This upper bound depends on local climate, large-scale atmospheric conditions and spatial variations
- Difficulty in estimating this upper bound due to the scarcity of data on extreme events

Goals:

- Combining statistical and physical modelling
- Taking into account spatial and temporal trends of the climate
- Generating large amounts of data at low cost for extreme events

Observed daily mean and maximum temperatures



Sources: Extreme Weather Watch, OpenData Paris

Statistical bound and physical bound

GEV upper bound

Let us define $T(t)$ the annual maximum temperature in year t.
Extreme-Value Theory suggests:

$$T(t) = \max_{1 \leq i \leq 365} T_i(t) \approx GEV(\mu, \sigma, \xi)$$

with $(\mu, \sigma, \xi) \in \mathbb{R} \times \mathbb{R}_+^* \times \mathbb{R}$.

Distribution function of a GEV distribution (Generalized Extreme Value)

$$G(x) = \mathbb{P}(T(t) \leq x) = \exp\left(-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right)$$

with x such that $1 + \xi\left(\frac{x-\mu}{\sigma}\right) > 0$.

$$\boxed{\text{For } \xi < 0, \quad 1 + \xi\left(\frac{x-\mu}{\sigma}\right) > 0 \quad \Leftrightarrow \quad x < \mu - \frac{\sigma}{\xi} =: B_{GEV}}$$

Statistical bound

For a given location, we model the annual maximum temperatures by:

$$T(t) \approx GEV(B_{stat}(t), \sigma, \xi)$$

with $B_{stat}(t)$ the statistical bound for year t estimated by maximimum likelihood

Plausible models must satisfy:

$$\mathbb{P}\left(T(t) > B_{stat}(t)\right) = 0$$

Possible violations of the upper bound:

- Model error: the temperatures $T(t)$ do not follow exactly a GEV distribution
- Parameter uncertainty

Physically-informed upper bound determined by atmospheric variables

According to physical laws derived by Zhang and Boos 2023 and Noyelle et al. 2024, the maximum surface temperature satisfies the following equation:

$$T_{\max} = T_{500} + \frac{L_v}{c_p}(Q_{sat}(T_{500}) - Q) + \frac{g}{c_p}(Z_{500} - Z_s)$$

with:

- T_{500} the air temperature at 500hPa
- Q the surface specific humidity of the air parcel
- Z_{500} the geopotential height at 500hPa

Physically-informed upper bound determined by atmospheric variables

Let us define $Y_j = T_{500}, Z_{500}$ or $-Q$. Let $Y_j(t)$ be the annual maximum value of Y_j for year t. Then, we assume:

$$Y_j(t) \approx GEV(B_j(t), \sigma_j, \xi_j)$$

with $B_j(t)$ the statistical bound for year t achieved by maximimum likelihood

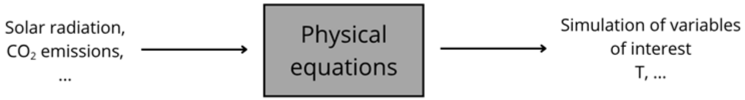
Therefore, we define an upper bound for T_{max} at year t as the following physical quantity:

$$B_{phy}(t) = B_{T_{500}}(t) + \frac{L_v}{c_p}(Q_{sat}(B_{T_{500}}(t)) + B_{-Q}(t)) + \frac{g}{c_p}(B_{Z_{500}}(t) - Z_s)$$

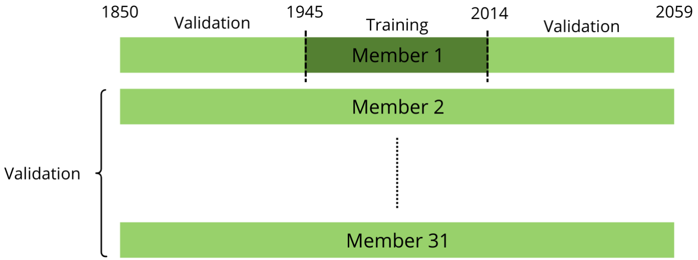
Data

Data simulated by a physical climate model

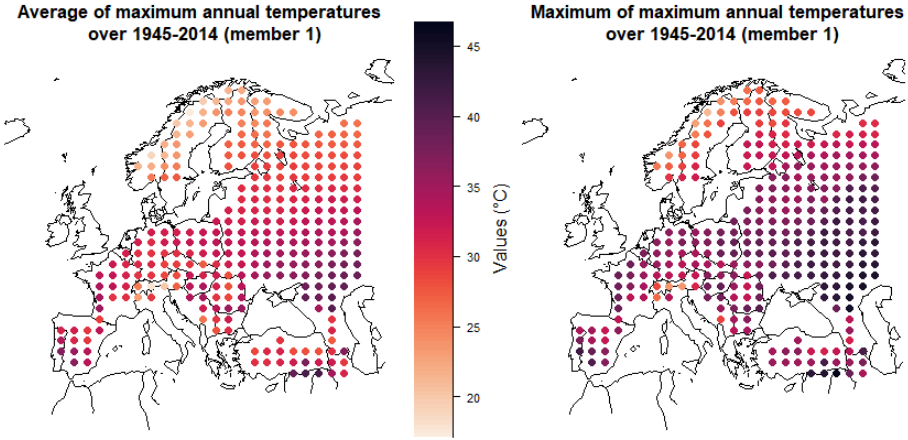
Climate model IPSL-CM6A-LR:



31 independent members:



Data simulated by a physical climate model



Results

Models

For a given location, the estimated parameters come from the model:

$$\begin{cases} B(t) \\ \sigma(t) = \sigma_0,\ \sigma_0 > 0 \\ \xi(t) = \xi_0,\ \xi_0 < 0 \end{cases}$$

Statistical upper bound:

$$B(t) = \hat{B}_0 + \hat{B}_1 \times \widetilde{GMST}(t)$$

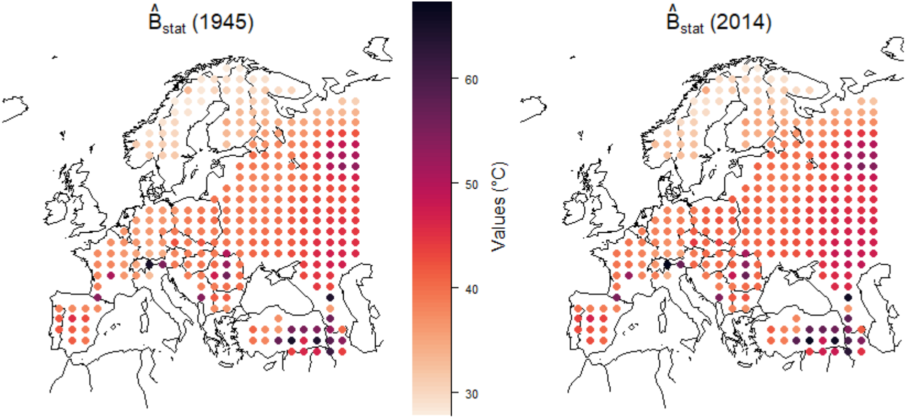
Physically-informed upper bound

$$\begin{aligned} B(t) = \quad & \hat{B}_{0,T_{500}} + \hat{B}_{1,T_{500}} \times \widetilde{GMST}(t) \\ & + \frac{L_v}{c_p}\left[Q_{sat}\left(\hat{B}_{0,T_{500}} + \hat{B}_{1,T_{500}} \times \widetilde{GMST}(t)\right) + \hat{B}_{0,-Q} + \hat{B}_{1,-Q} \times \widetilde{GMST}(t)\right] \\ & + \frac{g}{c_p}\left[\hat{B}_{0,Z_{500}} + \hat{B}_{1,Z_{500}} \times \widetilde{GMST}(t) - Z_s\right] \end{aligned}$$

defining $\widetilde{GMST}(t) = GMST(t) - \overline{GMST_{train}}$
(Global Mean Surface Temperature)

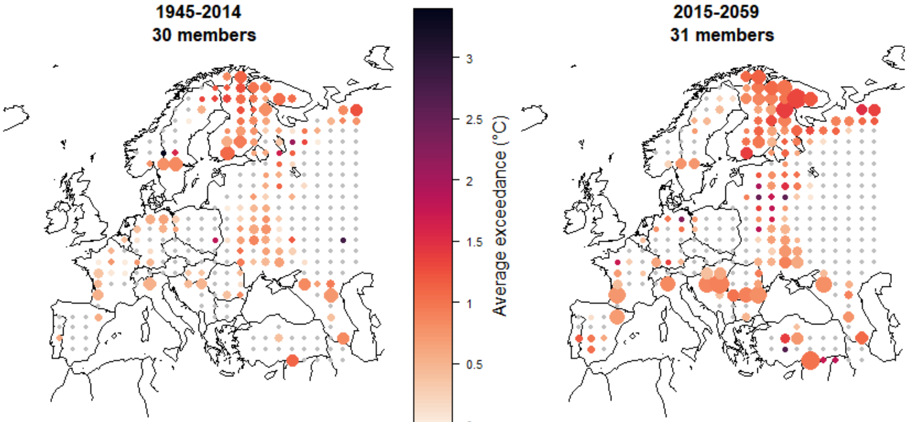
Statistical upper bound: results

MLE considering the nearby locations and penalizing ξ when close to 0



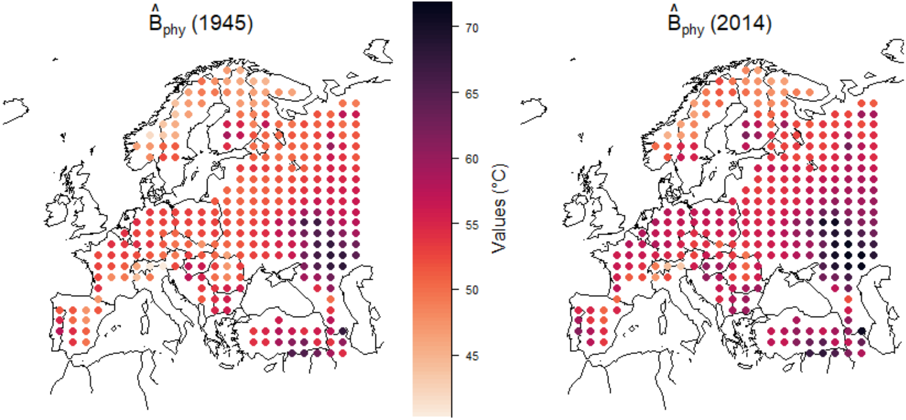
Statistical upper bound: exceedances

Point size: number of exceedances for the location studied



Physical upper bound: results

MLE considering the nearby locations and penalizing ξ when close to 0



→ No exceedance over 1850–2059 for all members of the climate model

Data simulation

Data simulation: annual maximum temperatures in Grenoble

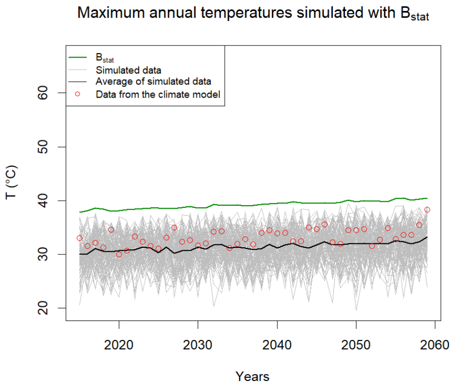
Generating data in Grenoble for years $t \in [\![2015; 2059]\!]$ from:

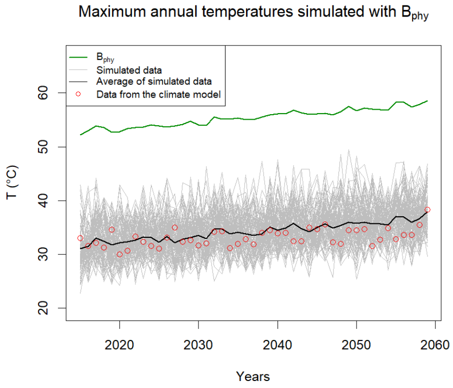
- $GEV(\hat{B}_{stat}(t), \hat{\sigma}, \hat{\xi})$
- $GEV(\hat{B}_{phy}(t), \hat{\sigma}, \hat{\xi})$

Data simulation: annual maximum temperatures in Grenoble

GMST from the member 1 of the climate model from 2015 to 2059

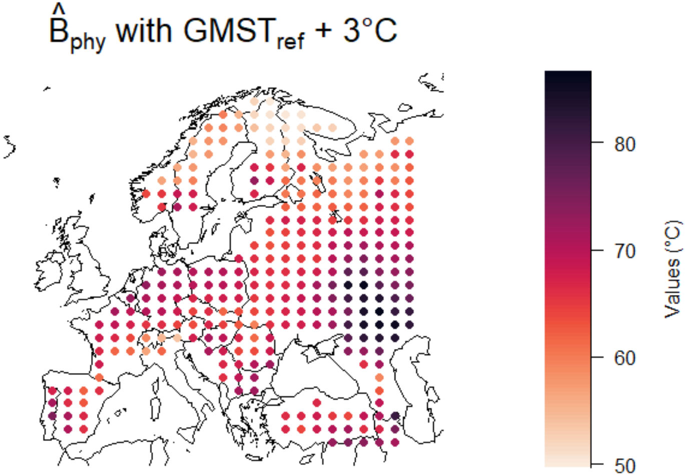
100 trajectories (grey)



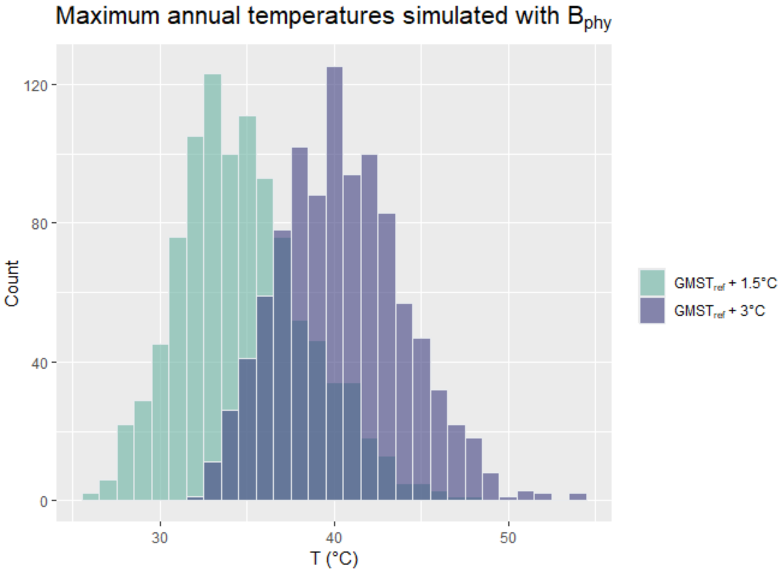


Data simulation: TRACC scenario for Europe

Scenario: Global warming will reach +3°C in 2100 compared to pre-industrial levels



Data simulation: TRACC scenario for Grenoble



Conclusion

Conclusion

In this work:

- Combining estimation of covariate bounds and physical equation gives greater upper limits
- No exceedance of the physics-informed upper bound while maintaining a reasonable goodness-of-fit
- Generating new data is easy and low-cost computing

Work in progress:

- Bayesian framework: defining more appropriate, principled priors for key parameters
- Estimate and propagate uncertainty of the physical bound by joint estimation of all parameters
- Spatial copula model for generating new data

References

Bücher, Axel and Johan Segers (2017). "On the maximum likelihood estimator for the Generalized Extreme-Value distribution". In: *Extremes 20, 839–872.*

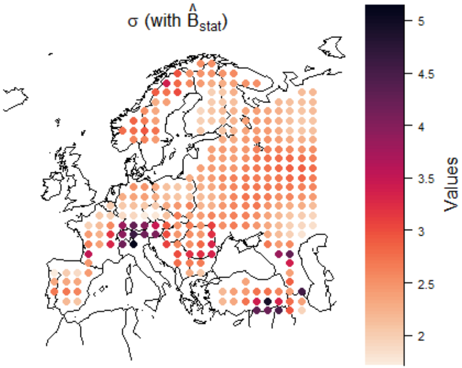
Coles, Stuart (2001). *An introduction to statistical modeling of extreme values*. Vol. 208. Springer.

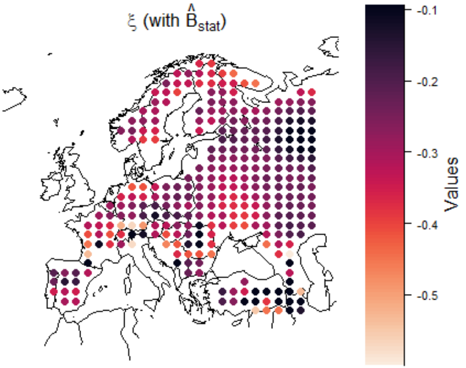
Noyelle, Robin et al. (2024). "Integration of physical bound constraints to alleviate shortcomings of statistical models for extreme temperatures". In: *HAL*.

Zhang, Yi and William R Boos (2023). "An upper bound for extreme temperatures over midlatitude land". In: *Proceedings of the National Academy of Sciences* 120.12, e2215278120.

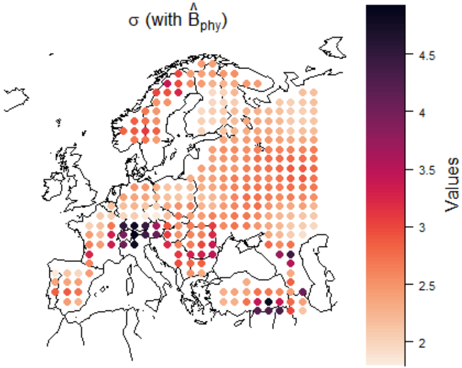
Appendices

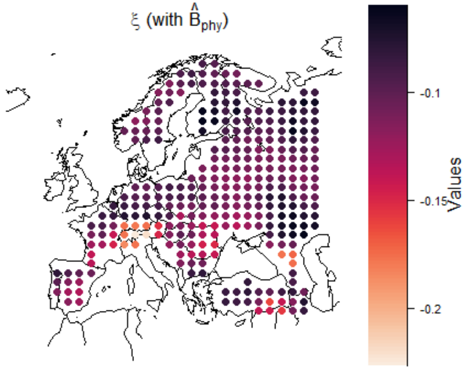
Statistical upper bound: scale and shape parameters





Physical upper bound: scale and shape parameters





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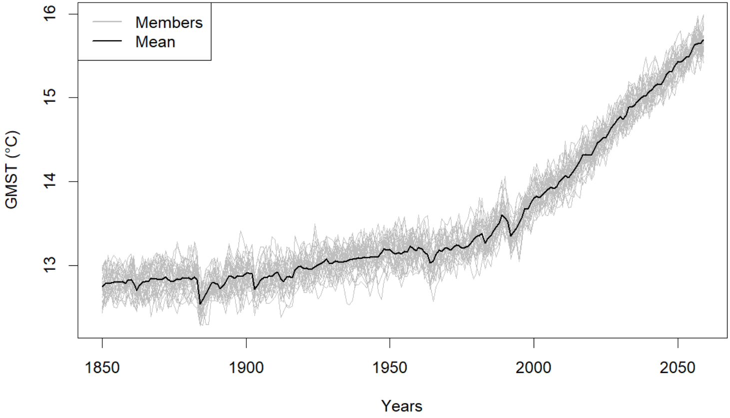

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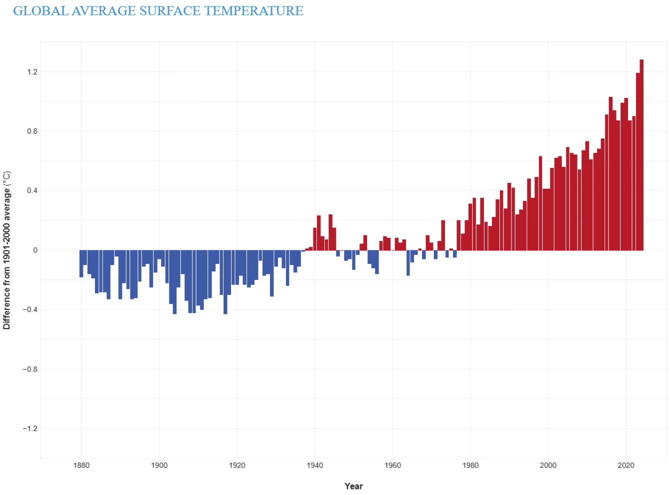

GLOBAL AVERAGE SURFACE TEMPERATURE
Difference from 1901-2000 average (°C)
1.2
0.8
0.4
0
−0.4
−0.8
−1.2
1880
1900
1920
1940
1960
1980
2000
2020
Year